Reply to Comment on "Roughness of Interfacial Crack Fronts: Stress-Weighted Percolaton in the Damage Zone"

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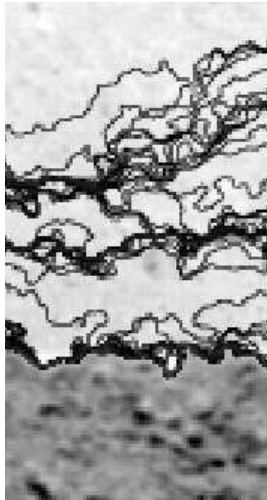*1361 Route des Lucioles, F–06560 Valbonne, France*

(Dated: February 2, 2008)

FIG. 1: Closeup of experimental fracture fronts from [3].

Alava and Zapperi (AZ) [1] question whether the fracture fronts we observe in [2] are self affine. We use Family-Vicsek scaling to determine the two scaling exponents α and z. AZ claim that this is not enough to determine whether the front is self affine and they go on to point to the presence of overhangs as their evidence of fractality rather than self affinity. In Fig. 1, we show details of *experimental* fracture fronts from Fig. 1 of [3]. There are significant overhangs, but these experimental fronts are self affine. In fact, as long as one allows a damage cloud to develop, overhangs are unavoidable. Family-Vicsek scaling implies non-isotropic scaling. A consequence is that the front width w scales with the width of the system L_x. Non-isotropic scaling is the defining property of self-affine surfaces.

AZ claim that the source of the L_x-dependency of the damage length scale l_y, Eq. (9) in [2], comes from the rescaling of the model's elastic constants when changing L_x. However, this rescaling is necessary to ensure that the elastic *properties* remain unchanged when the system size is changed under uniform loading conditions. It is caused by the non-local nature of the problem introduced by the Green function $\overline{G}_{i,j}$, Eq. (4) in [2]. Under uniform loading condition, Eq. (5) in [2] reads $u_i = u = \sum_j \overline{G}_{i,j} f_j = \sum_j \overline{G}_{i,j} b^2 \sigma$. Here b is the lattice constant. If u, local deformation, and σ, local stress, are to be independent of size, L_x and L_y, we must have that

$$\sum_j \overline{G}_{i,j} = \text{constant} . \tag{1}$$

Since $\overline{G}_{i,j} = \overline{G}_{i-j}$, we may for estetic reasons write $\sum_j \overline{G}_{i,j} = \text{constant} = \sum_{i,j} \overline{G}_{i,j}/(L_x \times L_y)$, as there is no dependency on the index i in Eq. (1). This was the way we chose to present it
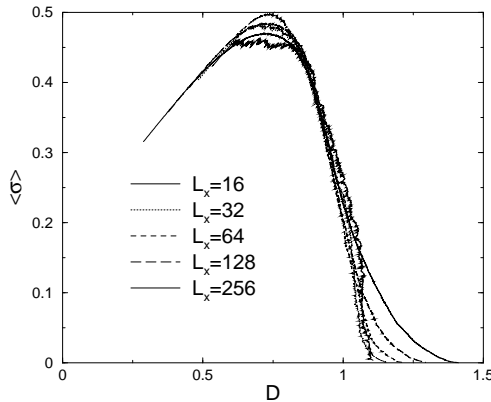
FIG. 2: Average stress $\langle\sigma\rangle$ as a function of imposed displacement D from virgin system to complete failure. The rescaling of $\overline{G}_{i,j}$, Eq. (1) and of the threshold distribution ensures that the curves collapse for different system lengths L_x and fixed $L_y = 128$.

in [2]. We point out again that both indices in the double sum $\sum_{i,j}$ run over $L_x \times L_y$ sites. We demonstrate the correctness of the rescaling in Fig. 2, where we show the collapse of the loading curves obtained for different system sizes after using Eq. (1). Only when the elastic properties have been rescaled as just described, one may proceed to use finite size scaling as done in [2].

We end this Reply by pointing out that Ramanathan and Fisher [4] measured numerically $\nu = 1.52 \pm 0.02$ using a very different model, which by construction cannot produce fractal fracture lines. This is in complete agreement with our model, which gave $\nu = 1.54$. The fracture roughness exponents, however, are very different.

[1] M. Alava and S. Zapperi, Phys. Rev. Lett. **92**, xxx (2004).

[2] J. Schmittbuhl, A. Hansen and G. G. Batrouni, Phys. Rev. Lett. **90**, 045505 (2003).

[3] K. J. Måløy and J. Schmittbuhl, Phys. Rev. Lett. **87**, 105502 (2001).

[4] S. Ramanathan and D. S. Fisher, Phys. Rev. B **58**, 6026 (1998).